ICT bringing mathematics to life and life to mathematics

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Abstract: *Many countries are seeking ways of encouraging more secondary school students to specialize in science, technology, engineering and mathematics (STEM). The talk will include examples of ICT-based activities used in some current school based projects to help make mathematics more exciting, relevant and challenging to young learners. These include the use of digital images and video clips to stimulate modeling with 2D geometry and algebra; the use of 3D geometry software to develop visualization and modeling in space; and the use of hand-held devices with data-loggers in capturing and analyzing experimental data.*

1. Introduction

This is a very exciting time in the development of the educational use of ICT (Information and Communication Technology) because of recent breakthroughs in technology which are making mobile computing devices ever smaller, powerful, robust, affordable and practicable [1]. Much of this is the result of new technologies developed within the Asian region, as foreseen in the "Shift happens" presentation: http://blogs.msdn.com/ukschools/archive/2008/02/18/shift-happens-and-freezing-frogs.aspx .

In the UK we have already seen considerable developments in the educational use of ICT to support classroom teaching of mathematics with nearly all teachers having access to laptops, data projectors and the Internet, and most also having the use of Interactive White Boards (IWB) and Virtual Learning Environment platforms (VLE). So we have given teachers the tools to do the job better – even if many choose not to use it much yet. The trouble is that the job itself is changing and needs to change – but we are far from being close to recognizing the ways in which that change impacts on a "traditional" subject like mathematics. In particular most students in secondary education (11-19) will not experience much, if any, hands-on use of computer technology during normal mathematics lessons – even though many schools have bought site licenses for mathematics software such as *Autograph, Cabri* and *Sketchpad*. Some schools use graphical calculators, and some are adopting TI-Nspire. Thus the major problem now being addressed is that of providing hands-on access for students to ICT in their normal mathematics lessons when and where needed. A recent conference held by the UK's National Centre for Excellence in the Teaching of Mathematics (NCETM) on the `Potential of ICT for Teaching and Learning mathematics' discussed these, and other important issues, such as continued professional development (CPD) for mathematics teachers [2]. The conference also included workshops illustrating a number of the major innovative school-based projects with ICT currently taking place in schools in England. This paper draws on a number of these projects.

There are other important changes taking place in the educational environment which have implications for the use of ICT in schools and colleges. One of the major changes is a general one across all subjects for students aged 11-19 with a rewritten curriculum emphasizing key concepts and processes and with less concentration on subject content. An implication of this is that schools

are expected to embed the new curriculum in a more `joined-up' fashion which emphasizes the links between subjects, as well as their relevance to the world outside education [3].

The particular change of greatest importance for this paper is the development of a national strategy to encourage more students to want to take qualifications, and to seek careers, in aspects of science, technology, engineering and mathematics – known as the STEM strategy for schools [4]. The context was a very important report from the UK government's then Chief Scientist, Sir Gareth Roberts, on the importance of these subjects for the future national economic well-being [5]. Clearly ICT is a universal tool in the practice of these subjects in the wider world – but also this paper sets out to exemplify how one important educational use of ICT can be to stimulate students' excitement and interest in STEM subjects in general, and in mathematics in particular.

The third major change is the abandonment of SATS (standardized assessment tests) at age 14 – which has involved many mathematics teachers in "teaching to the test". An example of a powerful way of giving teachers access to investigative approaches using a variety of mathematical tools is via the notebooks developed by the UK Mathematical Association for SMART exchange: http://exchange.smarttech.com/files/folders/math_association_uk/default.aspx .

2. Mathematics from a holiday snapshot

As we know, young people need no encouragement to engage with new technology – especially mobile phones, still/video images, surfing the web and social networking. So an obvious starting point is the use of a photograph – especially one they have taken themselves. Digital cameras have become one of the most widespread of domestic ICT products – even embedded in mobile phones. Oddly enough there was considerable speculation initially when they were first introduced about where their main market might lie. Their development was initially pitched at the requirements of real-estate agents and insurance companies who needed to keep large photographic archives in a way which would not deteriorate over time and which would be easy to retrieve. Together with webcams, digital scanners and picture searches from the Internet, it is now possible to create or find digital images of virtually everything under the sun. Some very useful sets of photographs intend for stimulus during mathematics lessons are available from Badsey Publications in the UK [6]. Photographs can be inserted as the background for graphs and/or constructions in software such as *Autograph, Cabri, Excel* and *Sketchpad*. For my example I will draw on a recent article in the Association of Teachers of Mathematics (ATM) journal `Mathematics Teaching' which was based on a few holiday photographs I took in Prague last summer after the ICTMT-8 conference in Hradec Králové [7]. I hope the ideas might stimulate those of you at ACTM 2008 in Bangkok to take home a few snaps as mathematical reminders of your visit.

Figure 2.1 A piece of cubist art in Prague

The photograph in Figure 2.1 shows a street lamp with an interesting geometric form, together with a human figure to provide an idea of scale. Can you estimate the volume and mass of the concrete post? Photographs can now be imported into a wide variety of mathematical software for further analysis (including *Excel*) – but in this case I have chosen to use geometry software. Using *Cabri II Plus* we can easily convert measurements taken from the image into their actual scale. These enable us to build, for example, physical scale models of objects using materials such as card. In this case, though, we can use *Cabri 3D* as a vehicle both to build the model and from which to take measurements. The basic building block is a frustum of a hexagonal pyramid. Ten of these form the main lamp post, and we can perform successive reflections of the basic polyhedron in the parallel faces to build the object. By scaling the measurements we can easily come up with a good approximation to the actual volume – and by finding out the density of concrete we can also make a good estimate of its mass.
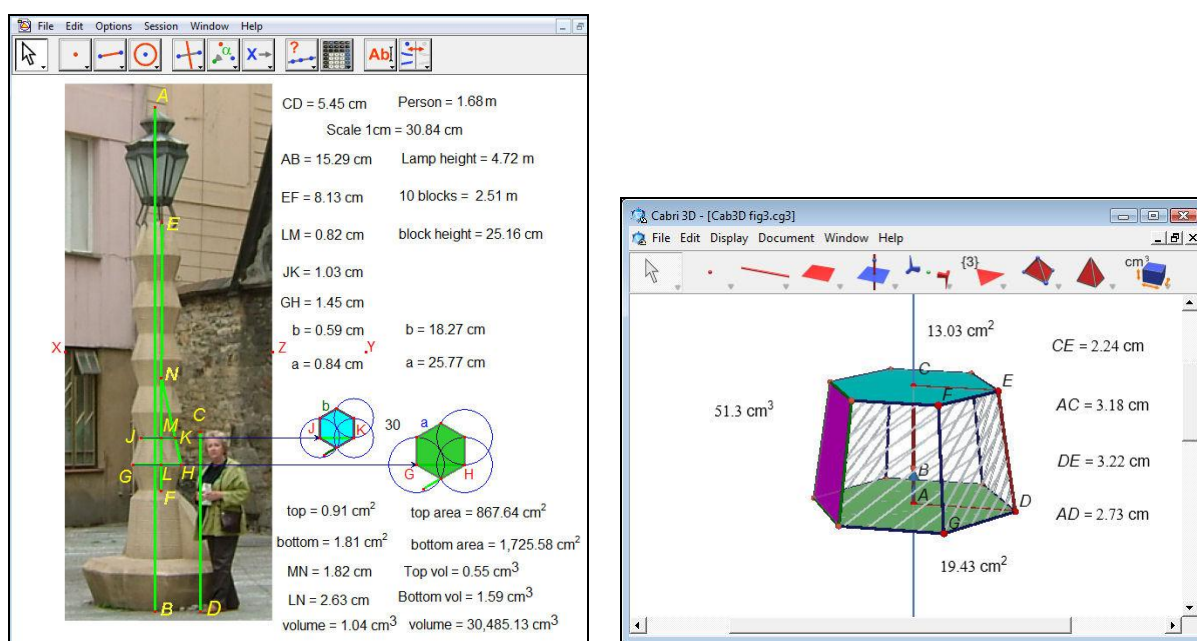

Figure 2.2 Modeling the geometry in 2- and 3-dimensions.

A nice source of ideas for modeling comes from mechanisms, such as the four-bar linkage. The still image taken by a photograph gives the necessary information from which to create a model, but the animation aspects of geometry software, such as *Cabri II Plus* and *Cabri 3D*, allow us – or rather, our students - to bring the static model to life. Another approach is to use photographs of chains, suspension bridges, water spouts etc. as sources of mathematical curves to which functions may be fitted using the graphing functions of dynamic geometry software – or in other graphing tools such as *The Mathematical Toolkit, Autograph* or *Microsoft Excel* [8], [9].

3. Mathematics from video clips

In some cases a still image will allow us to capture a great deal of data from a dynamical system – such as of the water-spout from the Merlion fountain in Singapore harbor. Knowing an actual distance, such as the spout above the water, we can fit a quadratic function and use it to calculate information, such as the velocity with which water leaves the spout. In other

circumstances, such as with a ball being thrown, there is not enough information in a still, but we can get a lot more from a short video clip - especially with free tools to edit and convert video formats, and to digitize and analyze them. Digital video cameras have also become more widely available – as well as being cheaper, smaller and more user-friendly. However, for most classroom use, the lowest resolution (e.g. 320 x 240) short "avi" video clip taken in the video mode of a digital camera or mobile phone is all that is required. Of course there are also on-line libraries, and DVDs, of video clips, of which Teacher Tube is one of the most relevant: www.teachertube.com. A children's playground with its dynamic apparatus like slides, swings and roundabouts is one good source of material – especially if the person being filmed can also be equipped with some data-logging equipment e.g. for acceleration. Another is the flight of a ball in a wide variety of sports and games. The important thing to bear in mind is that the camera needs to be held still, preferably using a tripod, and should not be used to track, or zoom on the moving object under question. Ideally, too, there should be an object of known dimensions within the field of view, to help in calibration, and the object being tracked should be far enough away to reduce effects of perspective and parallax. A different approach to the use of video is illustrated by the work of the UK Teachers' TV service which broadcasts digital free programs which can be viewed online and also downloaded by UK teachers who have registered on the site. This has rapidly become an unexpectedly effective CPD resource and there are many excellent programs to support the effective teaching of mathematics. In some instances they come together with the subject of this current paper. Thus, for instance, the program [10] demonstrates the use of data-capture from video clips of free throws at basket-ball as a source of data for modeling using quadratic functions. In the example shown, the teacher uses the US Open source physics software called *Vidshell* [11] to calibrate and annotate a video clip. The resulting still screen image is captured and pasted into geometry software, where the students choose appropriate origin and scales before modeling the trajectory of the ball with quadratic functions.
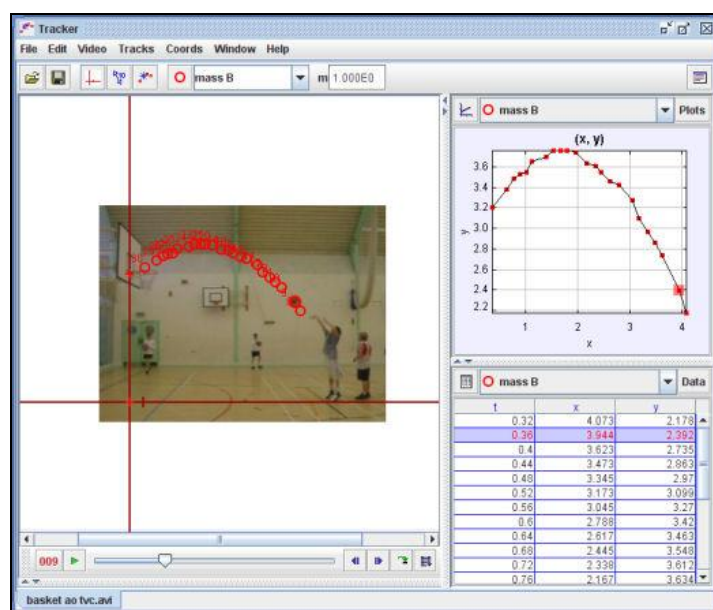

Figure 3.1 Modeling the basketball throw using *Tracker* video analysis.

The article [1] gives information about the more recent *Tracker 2* software [12], which is another US Open Source Physics (hence free) product – this time in *Java*. The article also describes

the use and source of the free *Mathematical Toolkit* software, developed jointly by Intel and the UK Mathematical Association, which includes facilities for graphing and video analysis [13].

4. Mathematical modeling and visualization in 3D

The UK secondary school curriculum contains a subject called "Design and Technology" (D&T) which involves aspects such as resistant media (wood, metal, plastic), systems & control (electronics, mechanisms, hydraulics), food technology and textiles – with an emphasis on design, testing and making. A major recent government initiative has been to introduce CADCAM in schools' D&T departments: www.cadinschools.org. In math, students get very little 3D work – maybe a few questions where they have to identify and solve a right-angled triangle within a 3D diagram. Yet we live in a 3D world which many people have to control, or simulate, as part of their daily lives. The availability of powerful 3D modeling tools, particularly *Cabri 3D,* which won a coveted BETT award in 2007, now enables mathematics teachers to develop approaches to using ICT to encourage pupils' powers of visualization and modeling in 3D, as recommended in the Royal Society's Geometry report [14]. The Mathematical Association and the Association of Teachers of Mathematics, supported by the British Educational and Communications Technology Agency (Becta) have been working with a group of teachers to produce case studies of the use of ICT to support the teaching of "hard to teach" aspects of the secondary curriculum. These case studies will be available through the NCETM website, and three of them are featured on a Teachers' TV program scheduled for first broadcast on November 10th 2008 [15]. A particularly powerful feature in *Cabri 3D* to aid visualization is the ability to fold back the sides of a 3D solid shape to show its net and to reveal its interior.

Figure 4.1 Excerpts from Teachers' TV `Hard to teach Mathematics'

Oldknow and Tetlow reported on other aspects of the use of 3D modeling tools for mathematics in their contribution to the 2007 ICTMT-8 conference [16]. Again we can relate this to the technological world the students inhabit by building simulations and models with 3D geometry software – showing, for example, central orbits and spherical triangles.

5. Modeling motion with real data

We have seen one example of capturing data from a moving object using video analysis. Another significant development in the educational use of ICT has been in the area of data-logging and sensors – which are now an integral part of the English science curriculum. There are many experiments which do not require specialized apparatus and which can be performed safely inside a

mathematics classroom, or outdoors. One of the simplest, and most powerful, applications of data-logging for mathematics is capturing time and distance data for a moving object using a `motion detector' such as the TI CBR2. This device costs around £75 and collects data on the distance to the nearest object. It is ideal for producing displacement-time graphs. You can see this in use for whole class teaching on Teachers' TV: http://www.teachers.tv/video/154 and also a review at: http://www.teachers.tv/video/4872. In this example we have captured data on a student walking away from the CBR at a fairly state pace, then stopping briefly before coming back a bit faster. We don't know which way s/he was facing, but of course those present in the room have captured a "video" in their memory against which they can compare the distance *v* time graph.
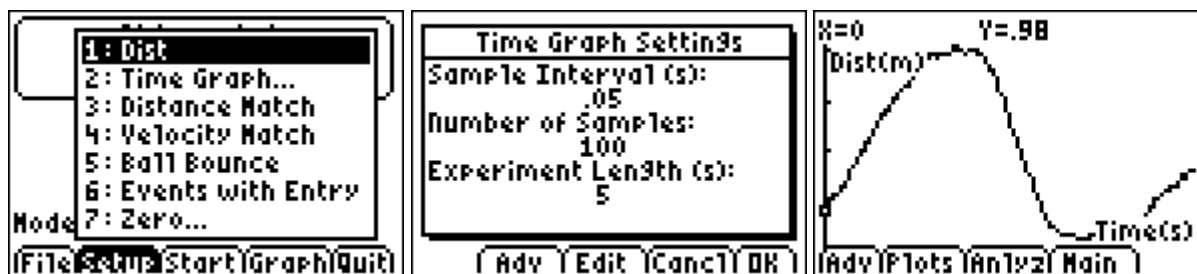

Figure 5.1 Capturing data for a distance time graph

Figure 5.2 *TI-SmartView* in the classroom; TI-84 Plus and TI-Nspire for outside

Ideas for cross-curricular data-logging and modeling at Key Stage 3 (11-14) can be found in [17]. Oldknow and Taylor reported on a number of data-logging activities to support cross-curricular work (11-19) in the ICTMT-8 conference [18]. The CBR can also be used to track the rebounds of a bouncing ball. In this case we are looking at a distance-time graph of a ball which is (more-or-less) bouncing on the spot, and not a trajectory in space like the basket-ball throw.
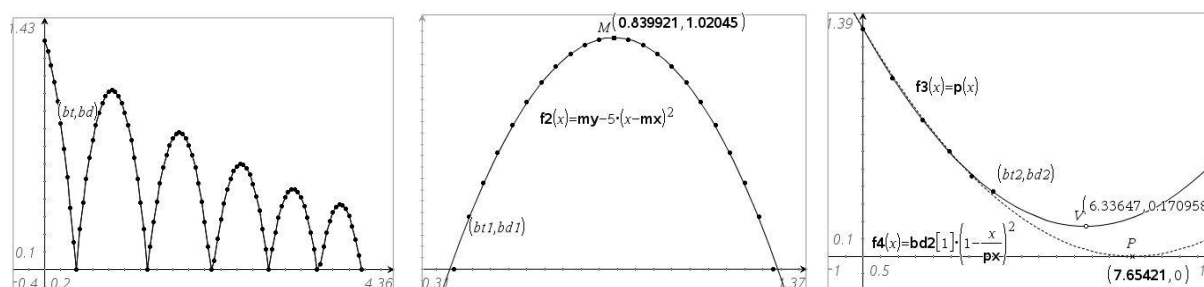
Figure 5.3 Bouncing ball data analyzed with *TI-Nspire*

We can fit a quadratic function to any one of the complete bounces – either by trial-and-improvement, as shown, or by using a built-in "best-fit" (or "regression") function – which is available in the latest version of *Excel*, and in many good mathematical software packages, such as *TI-Nspire*, which was used for the images above. Another interesting (but harder) problem is to find a function to model the successive maximum points on the graph. At first sight they look like an exponential decay – but surprisingly they are well-fitted by a quadratic function! Nor by the "best-fit" graph $y = f3(x)$, which has a minimum at $y = 0.17$ - which would suggest that all the bounces would be at least 0.17m high! But as each bounce takes a shorter time, there is the possibility that an infinite number of bounces could take place in a finite time. The function $f4(x)$ is a quadratic function which passes through the point where $t = 0$ and the point P on the t-axis. By sliding P we can make the function pass as close as possible to the data points – and see that all bounces will have died out in about 7.5 seconds.

We have seen two different approaches to data-capture – one using a sensor (in this case a motion detector) and the other using a video clip. Wouldn't it be great to put them together - especially if we didn't need a wire to connect the data-logger to a computer or hand-held device! One way to do this is to set the logger to capture and store data automatically, and to download it to the device after the experiment. Another is to send the data in real-time, wirelessly, to the device using e.g. a Bluetooth connection.

Vernier's *Logger Pro 3* software supports video capture alongside other forms of experimental data-logging. This means, for example, that you can choose whether to let the software control real-time video capture from, say, an attached web-cam while simultaneously logging data such as force and acceleration from a spring/mass experiment, or whether to video it independently and subsequently to synchronize the resulting video clip with the displayed data. Vernier also produces a Wireless Dynamics Sensor System (WDSS) which can be connected using Bluetooth so that data can be logged wirelessly in real time. This has an altimeter, 3 accelerometers and a force sensor. The picture in Figure 5.4 is a still from a video clip of an elephantine "bungy jumper" connected to a strong spring via the Vernier WDSS. The wireless kit is detected by *Logger Pro*, and used to capture and display data such as acceleration and force. The video clip is imported and adjusted so that a given frame corresponds to a particular reading, such as when displacement is at a minimum or maximum. The corresponding video, graphical and numerical information are shown in the following *Logger Pro* screenshot. Needless to say *Logger Pro* also contains tools to fit your own functions to graphs and to compute best fits.
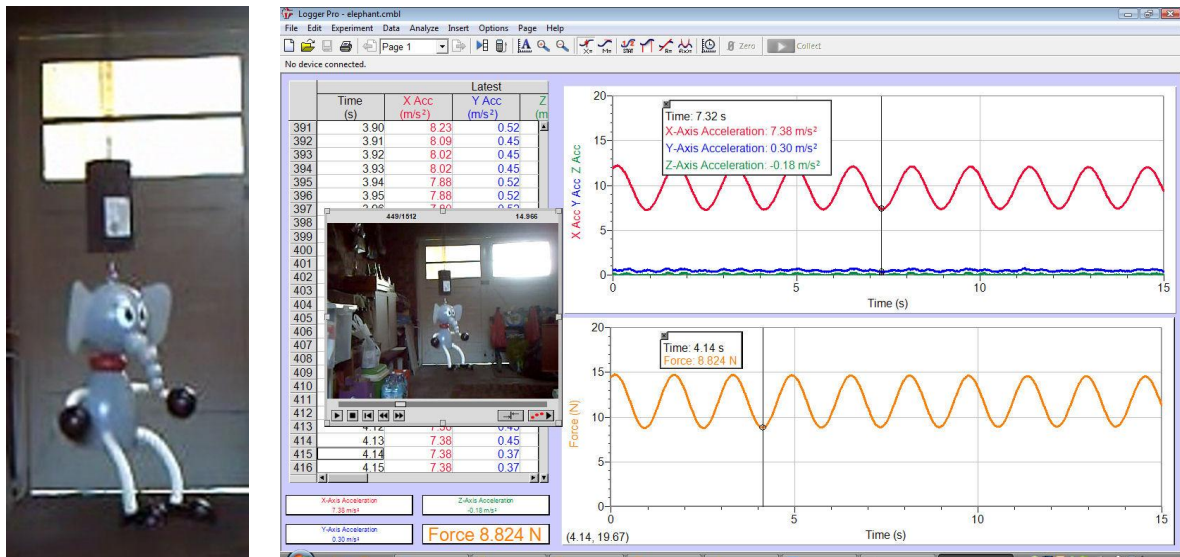
Figure 5.4 a spring-mass system wirelessly modeled with *Logger Pro*

Texas Instruments' new *TI-Nspire* hand-helds and software work directly with USB probes such as the CBR to capture data from experiments such as the oscillating elephant. The data-handling and analysis software greatly facilitates explorations of models – such as with the phase0plane diagram illustrated below.
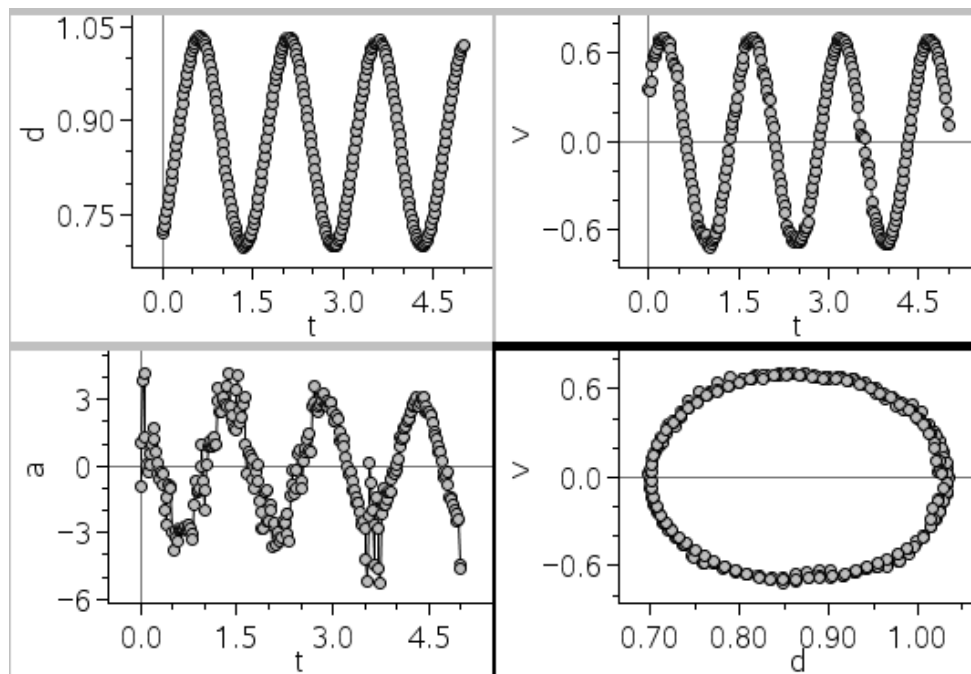

Figure 5.5 *TI-Nspire* and CBR showing a variety of dynamic graphs

6. Putting the ideas together – is it rocket science?

We looked for ideas of activities which would provide opportunities to integrate both the STEM subjects and some of the varied mathematical approaches using ICT illustrated here. Rockets was a popular theme. 2007 was the first year of a new national competition for UK schools run by the UK Rocket Association (UKRA) www.ukayroc.org.uk/ - the UK Aerospace Youth Rocket Challenge. A team of Year 8 pupils (12-13) entered for the challenge - to design, build and fly a rocket which would reach a maximum altitude of 260m and deliver an unbroken egg to earth in 45 sec. This year the school entered two teams, one from Y8 and one from Y9. Both qualified for the finals, which took place in such foul weather that they were unable to make a final set of launches, and instead had to make team presentations to a panel of UK rocket scientists! So while STEM is at the heart of this competition the students also have to have excellent ICT and communication skills. You can see the school's video at: www.croftonschool.co.uk/videos/rocket/rocket-project.html and the teams' presentations at www.croftonschool.co.uk/videos/rocket/rocket-presentations.html . While this is an excellent cross-curricular project it is rather expensive in terms of materials, equipment and time. It also requires considerable care from the safety viewpoint. So we needed to find a simpler experimental vehicle. Also the mathematics of powered flight proved too difficult to model at an elementary level – so we needed to combine projectile motion with ballistic rockets – hence STOMP!
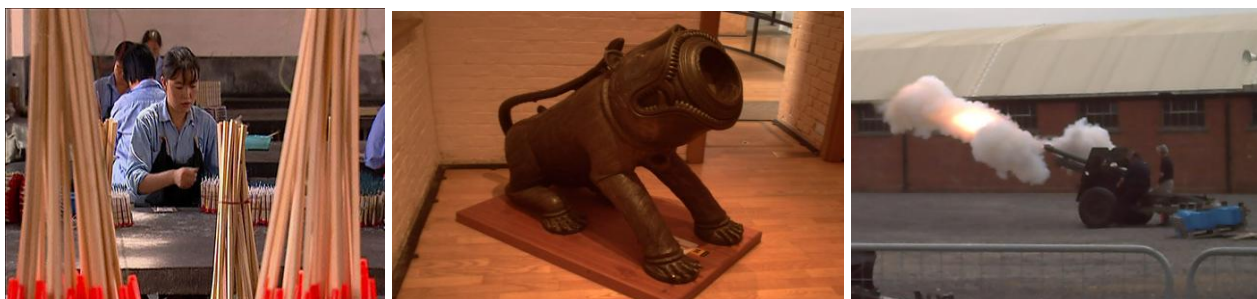

Figure 6.1 fireworks, a mortar and a field gun – all launched using gunpowder

The best solution we have found so far is a simple air-powered plastic rocket called STOMP. Students jump on the plastic box to launch the rocket. They have a simple data-collection kit consisting of a stop-watch, a protractor and a tape measure. They record their results in the field and can analyze them using hand-held technology such as TI-84 Plus or TI-Nspire. On return to the classroom they can enter their data into software models e.g. in *Excel*, dynamic geometry or *TI-Nspire* software. Using expressions for average velocity and acceleration they derive Galileo's formulae and use them, with recorded values of launch angle, distance travelled and time taken, to compute values for *g*, the launch velocity and the maximum height and hence plot the trajectory.
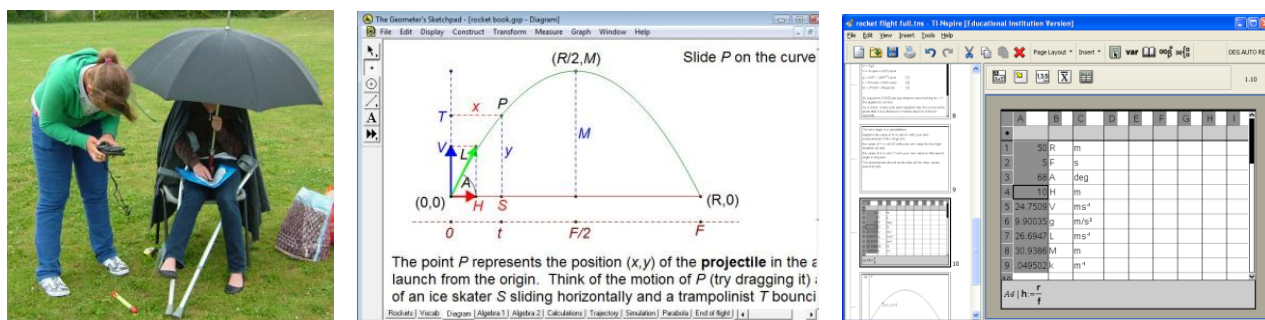
Figure 6.2 a simpler, cheaper and safer approach – but still a lot of fun, even in the rain
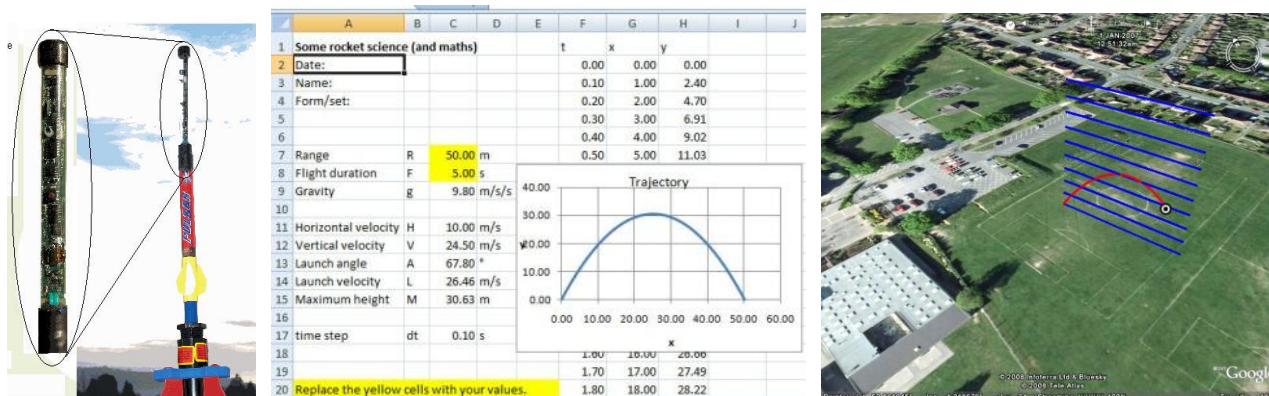

Figure 6.3 the Sciencescope rocket gathers real data for analysis and display

From the trajectory they can even find approximations for the distance travelled. The analysis of the STOMP air rocket data depended on using a simplified model from Newtonian mechanics in which spin, wind and air resistance are ignored. An excellent vehicle for testing the validity, or otherwise, of such a model is to use a rocket equipped with a nose cone containing sensors such as height, pressure and acceleration. The Sciencescope company has produced just such a product – which comes together with a more high-powered rocket powered by compressed air: http://www.sciencescope.co.uk/rocketlogger.htm. The data are downloaded following a flight, and can be exported to *Excel* for analysis and display. The data can also be saved in a format which produces a simulated video of the launch in *Google Earth*! By comparing the recorded trajectory against the Galileo/Newton model students can see to what extent factors like air-resistance can safely be ignored. They can also explore possible models for air resistance as powers of velocity.
Suddenly it's really caught on. But we also need to inform students of contexts which relate rockets to their own life – e.g. satellite launching, orbits and dish aerial – without which they would not have mobile phones, social networking on the Internet, GSP navigation or satellite television. So the geometry of the parabola (e.g. in the development of the telescope, in producing light beams etc.) is as important as algebra of quadratic function.

7. Conclusion

The paper has given examples of ICT-based activities developed recently in a number of UK secondary schools. They use readily available software and other ICT tools, and have proved inspiring for both students and teachers. However the teachers have been prepared to take risks –

and not every activity attempted has been met with complete success. One important dimension added by the teachers has been to relate the activities to the world familiar to their students. While most of them realize that their mobile phones have digital cameras, few realize that these phones would not work were it not for the rocket science which put the communication satellites in orbit - or that is the geometric properties of the parabola that enable them to watch satellite TV! ICT cannot displace the mathematics teacher, but I hope to have exemplified how it can make him or her more effective in winning the hearts and minds of the next generation of technologists.

We have the technology. Much of it is already in the hands of the learner – more soon will. They have the skill to put it to any use. As educators we need to find ways to unlock those skills so that they increase students' learning about the world of mathematics, science and technology; their curiosity about how the objects around them work; and the excitement, challenge and pleasure they get from relating the first with the second. Thank you – and good luck.

Acknowledgements

References

[1] Oldknow, A. (2008). *It's 2008 - So what you got to offer, then? - Using ICT to put learners in touch with mathematics.* National Centre of Excellence in the Teaching of Mathematics, London, UK: http://www.ncetm.org.uk/files/251777/Becta+article+AO.pdf .

[2] Oldknow, A. (2008). *Conference report – the potential of ICT for teaching and learning mathematics.* National Centre of Excellence in the Teaching of Mathematics, London, UK: http://www.ncetm.org.uk/files/266270/Report+by+Adrian+Oldknow.doc .

[3] QCA. (2008). *Programmes of study: Key stage 3 & 4 mathematics.* Qualifications and Curriculum Authority (QCA), London, UK: http://curriculum.qca.org.uk/key-stages-3-and-4/subjects/mathematics/index.aspx .

[4] DCSF. (2008). *£140m Boost To Science And Maths Teaching In Schools.* Department for Children, Schools and Families (DCSF), London, UK: http://www.dcsf.gov.uk/pns/DisplayPN.cgi?pn_id=2008_0017 .

[5] HM Treasury, (2002) *SET for Success - Final Report of Sir Gareth Roberts' Review*. HM Treasury, London, UK: http://www.hm-treasury.gov.uk/media/6/F/chap_02.pdf .

[6] Philips, R. (2007). *Problem Pictures*. Badsey Publications, Badsey, UK: http://www.badseypublications.co.uk/ .

[7] Oldknow, A. (2008). *Cubism and Cabri*. Mathematics Teaching 206, January 2008, the Association of Teachers of Mathematics (ATM), Derby, UK: http://www.adrianoldknow.org.uk/ATM-MT206.pdf .

[8] Oldknow, A. (2003). *Geometric and algebraic modelling with dynamic geometry software*. Micromath V19N2, ATM, Derby, UK: http://www.atm.org.uk/mt/micromath/mm192oldknow.pdf .

[9] Oldknow, A. (2003). *Mathematics from still and video images.* Micromath V19N2, ATM, Derby, UK: http://www.atm.org.uk/mt/micromath/mm192oldknowa.pdf .

[10] Teachers' TV. (2007). *Hard to teach – secondary mathematics - quadratic function.* Teachers' TV, London: http://www.teachers.tv/video/19119 .

[11] Davis, D.V. (2000). *Webphysics - Vidshell*. New Hampshire Community Technology College, Brunswick, NH, USA: http://www.webphysics.ccsnh.edu/vidshell/vidshell.html .

[12] Brown, D. (2008). *Tracker – video analysis and modeling tool*. Open Source Physics, Cabrillo College, Aptos, CA, USA: http://www.cabrillo.edu/~dbrown/tracker/ .

[13] The Mathematical Association (MA) & Intel. (2008). *The Mathematical Toolkit.* London Grid for Learning, London, UK: http://lgfl.skoool.co.uk/content.aspx?id=657 .

[14] Royal Society. (2001). *Teaching and learning geometry 11-19*, The Royal Society, London, UK: http://royalsociety.org/document.asp?id=1420.

[15] Teachers' TV. (2008). *Hard to teach – secondary mathematics using ICT.* Teachers' TV, London: http://www.teachers.tv.

[16] Oldknow, A. & Tetlow, L. (2008). *Using dynamic geometry software to encourage 3D visualisation and modelling*, eJMT V2N1, Radford University, VA, USA

[17] Oldknow, A. & Taylor, R. (1998). `*Data-capture and modelling in maths and science'*, NCET, Coventry, UK: www.adrianoldknow.org/datacapture.pdf.

[18] Oldknow, A. & Taylor, R. (2008). *Mathematics, science and technology teachers working collaboratively with ICT*, eJMT V2N1, Radford University, VA, USA.